Further Remarks on Final Exam Answers*

no-Ponzi conditions: It was common for people, sometimes even after writing down examples of the conditions correctly, to say that a no-Ponzi condition required "non-negative net worth at infinity". The most obvious reading of this phrase would suggest that it implies $W_T \to \bar{W} < 0$ is ruled out, but obviously when this condition holds, $\beta^T W_T \to 0$ holds also. So, though transversality prevents wealth from rising too fast as $t \to \infty$ and no-Ponzi conditions prevent debt from rising too fast as $t \to \infty$ (and most people got this right) they do not prevent $W_T \to \pm\infty$ in general. (TVC's may do so in some particular models.)

Question 4: **b:** A surprising number of people asserted that the parameter which, if set to zero, would make the model show neutrality is θ, the price elasticity of demand. Some thought this made the model competitive. But $\theta = \infty$, not $\theta = 0$, is the competitive case. With $\theta = 0$, demand is price inelastic, firms would like to get $P/\bar{P}$ has high as possible, and there is no steady state for the model. The model certainly does not show neutrality in this case.

c: Many answers ignored government policy altogether, suggesting that monopolistically competitive pricing behavior determines the price level. But the question was asking for a contrast between this model and one containing money. Even if money were present, the firm pricing behavior would be the same. It is only the government budget constraint that would take on a different character. To explain what replaces money as the determinant of the price level, one must discuss the GBC and fiscal and interest rate policy.

d: The answer I posted, I realized as I graded the exams, forgot to mention two important classes of conditions that are needed to determine equilibrium. One is the TVC's for the firm and the individual. For the individual the TVC is

$$\limsup_{t\to\infty} \beta^t E\left[-\frac{dB_T}{P_T C_t}\right] \le 0\,, \tag{1}$$

and since $B_t \ge 0$ is imposed and $\lambda_t > 0$ for all t, this is equivalent to

$$\lim_{t\to\infty} \beta^t \frac{B_t}{P_t C_t} = 0\,. \tag{2}$$

For the firm the TVC is

$$\limsup_{t\to\infty} \beta^t E\left[-\left(\zeta_t \frac{L_t^\alpha}{\bar{P}_t} - 2\xi\zeta_t \frac{P_t - P_{t-1}}{\bar{P}_{t-1}^2} - \nu_t\theta \frac{P_t^{-\theta-1}}{\bar{P}_t^{-\theta}}\right) dP_T\right]\,. \tag{3}$$

Again, since $P_T > 0$ and $\zeta_t > 0$ in any equilibrium, We can replace dP_T in this expression by $-P_T$. The TVC's as we have used them are only sufficient conditions, so a path for the economy that doesn't satisfy them could conceivably still be an equilibrium, though that is unlikely.

The other class of conditions I omitted was $P_t \equiv \bar{P}_t$, $L_t \equiv \bar{L}_t$. This is almost trivial, and I expected people to make the substitution in writing down their answers. However, most people did not make the substitution, and some wrote as if these conditions might not hold, so a slight amount of credit was deducted for failing to mention them.

Question 5 (!): This question's answers were chastening for me. Stuff I thought I had taught the class nearly everyone seemed not to know. Nearly everyone either cautiously avoided stating whether it was existence or uniqueness that failed when the number of roots didn't match the number of endogenous errors, or else stated exactly the opposite of the correct answer. Some people who could back up their answers from first principles even arrived at the wrong conclusion. The fallacious reasoning goes as follows:

> If there are too many (more than two) unstable roots, then I could solve forward using any two of them, and each pair would give me a different way to solve for η from knowledge of ε. This must be non-uniqueness. And if there are two few unstable roots, I won't have enough equations to solve for both η's from z. This must be non-existence.

But in fact too many unstable roots generally implies non-*existence*. A solution must satisfy all equations at once. If different subsets of relations in the model have different solutions, there is no one answer that satisfies all equations in the model.

And if there are too few unstable roots, there are not enough equations to mechanically solve for both η's from the zero or one forward-solved equations. But if there are $m < 2$ unstable roots, one can set $n - m$ elements of η arbitrarily and solve for the remaining ones. Because of the arbitrariness, this implies non-*uniqueness* of the solution.

c: Hardly anyone understood what this part was getting at. It was meant to let you point out the analogy between this model and standard models with money and bonds. As we discussed in class, in models with money and bonds it is generally possible to eliminate bonds from the system and ignore the government budget constraint. This does not mean the government budget constraint and bonds are not there. *We are using a smaller system of equations derived from the FOC's of a system that included bonds.* So the smaller equation system is not a different model, just a set of equations implied by the model. This means that to the extent the smaller system delivers conclusions about existence and uniqueness different from the larger system, the smaller system is wrong, as it is an incomplete characterization of the model. In models with money and bonds, it is common in macroeconomic theory to ignore the government budget constraint, thereby

implicitly assuming that fiscal policy has a form that validates conclusions about existence and uniqueness from a system ignoring the budget constraint. But with other fiscal policies—for example primary surplus set exogenously—one simply gets the wrong conclusions if the GBC is ignored.

Though the question did ask you to discuss the two "versions" of "the" model, and it should have been clear that the smaller set of equations was derived from the same FOC's, and hence the same model, as the larger set of equations, the question should have been worded more clearly. Many people interpreted the smaller set of equations as referring to a different model, in which there are no bonds and no taxes.